Magnetic Helicity and Planetary Dynamos. John V. Shebalin, NASA JSC, Mail Code KR, Houston, TX 77058.

Introduction: A model planetary dynamo based on the Boussinesq approximation along with homogeneous boundary conditions is considered. A statistical theory describing a large-scale MHD dynamo is found, in which magnetic helicity is the critical parameter.

Model System: The theory of planetary dynamos remains an open and active research area [1]. Here, an unforced, incompressible, turbulent, buoyant magnetofluid, constrained by concentric inner and outer spherical surfaces, is considered. This model system is analyzed using the Boussinesq approximation along with homogeneous boundary conditions, i.e., temperature fluctuation τ and normal components of the velocity $\mathbf{u}$, magnetic field $\mathbf{b} = \nabla\times\mathbf{a}$, vorticity $\omega = \nabla\times\mathbf{u}$ and electric current $\mathbf{j} = \nabla\times\mathbf{b}$ are zero on the boundaries ($\mathbf{a}$ is the magnetic vector potential). This choice of boundary conditions allows for a set of Galerkin expansion functions that are common to the physical fields τ, $\mathbf{u}$, $\mathbf{b}$, ω and $\mathbf{j}$ to be utilized.

Analysis: This model dynamical system represents magnetohydrodynamic (MHD) turbulence in a spherical domain and is analyzed in by methods similar to those applied to homogenous MHD turbulence [2,3]. A statistical theory of ideal (no dissipation) MHD turbulence is found, analogous to that in the homogeneous case, including the prediction of coherent structure in the form of a large-scale quasistationary magnetic field. This MHD dynamo depends on broken ergodicity [4], an effect that is enhanced when total magnetic helicity is increased relative to total energy. When dissipation is added and large scales are only weakly damped, quasiequilibrium may occur for long periods of time, so that the ideal theory is still pertinent on a global scale. Over longer periods of time, the selective decay of energy over magnetic helicity further enhances the effects of broken ergodicity. Thus, broken ergodicity is the essential mechanism and relative magnetic helicity is the critical parameter in this model MHD dynamo theory.

Qualitative Discussion: The analytical details that will be presented at the conference are somewhat technical and require more space than is available here, so a qualitative and heuristic discussion will be given instead. MHD turbulence is very different from fluid turbulence, where there is no electrical conductivity. In fluid turbulence, there are two ideal integral invariants, energy $E = ½\langle u^2\rangle$ and kinetic helicity $H_K = ½\langle\mathbf{u}\cdot\omega\rangle$, where $\langle Q\rangle$ denotes the integral of the quantity Q over the volume under consideration. In the case of rotating MHD turbulence, the ideal invariants are energy $E = ½\langle u^2+b^2\rangle$ and magnetic helicity $H_M = ½\langle\mathbf{a}\cdot\mathbf{b}\rangle$. In the case of fluid turbulence, both E and H_K have direct cascades to higher wave number k (i.e., smaller scales), while in MHD turbulence, E has a direct cascade to high-k, while H_K has an *inverse* cascade to low-k (see, for example, [5]). The net result of this is that magnetic energy appears in the largest scales of MHD turbulence. In addition, what we found numerically is that the large-scale MHD structures are *coherent* [2] and, finally, a theory to explain exactly why this occurred [3].

The magnetic field may be viewed heuristically as having a magnetic tension along a field line (like a stretched rubber band) and a magnetic pressure normal to a field line. In the ideal limit, magnetic field lines are “frozen into the plasma” [6], meaning that the magnetofluid can slip along a field line but is tied to the fluid for any motion perpendicular to the field line. In the case of MHD turbulence, this implies that if a magnetofluid is stirred vigorously, so that the magnetic field lines are highly convoluted and bent, then the magnetofluid will relax through straightening and expanding its magnetic field lines so as to fill itself with a large-scale structure. This, of course, is an approximation, but one that is increasingly better the larger the kinetic and magnetic Reynolds numbers.

Conclusion: Large-scale planetary magnetic fields are theorized to originate in a liquid core due to MHD processes. Successful numerical simulations the of geodynamo based on the Boussinesq approximation [7,8] give strong support for this belief. Qualitative reasoning (above) suggests that large-scale magnetic structure is inherent in MHD turbulence. What will be given in this presentation is a new and novel theoretical explanation, for a model system closely related to those employed previously, as to why this magnetic structure is both large-scale and coherent. In this theory, magnetic helicity is the critical parameter and “broken ergodicity” is the essential mechanism.

References:

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